Secondary Structure Prediction for Aligned RNA Sequences

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Most functional RNA molecules have characteristic secondary structures that are highly conserved in evolution. Here we present a method for computing the consensus structure of a set aligned RNA sequences taking into account both thermodynamic stability and sequence covariation. Comparison with phylogenetic structures of rRNAs shows that a reliability of prediction of more than 80% is achieved for only five related sequences. As an application we show that the Early Noduline mRNA contains significant secondary structure that is supported by sequence covariation.

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Introduction

Most functional RNA molecules exhibit a characteristic secondary structure that is highly conserved in evolution. Examples include tRNAs, the rRNAs (5 S, 16 S, as well as 23 S), RNaseP RNA, the RNA component of signal recognition particles (srpRNA), tmRNA, and group I and group II introns. This list can be extended by numerous families of artificially selected catalytic RNAs.

It is of considerable practical interest therefore to compute efficiently the consensus structure of a collection of such RNA molecules. Such an approach must combine the phylogenetic information contained in the sequence covariations as well as thermodynamic stability of molecules. Combinations of phylogenetic and thermodynamic methods for predicting RNA secondary structure fall into two broad groups: those starting from a multiple sequence alignment and algorithms that attempt to solve the alignment problem and the folding problem simultaneously. The main disadvantage of the latter class of methods is their high computational cost, which makes them unsuitable for long sequences such as 16 S or 23 S RNAs. Most of the alignment based methods start from thermodynamics-based folding and use the analysis of sequence covariations or mutual information for post-processing. The converse approach is taken in, where ambiguities in the phylogenetic analysis are resolved based on thermodynamic considerations.

In this contribution we describe a combined approach that integrates the thermodynamic and phylogenetic information into a modified energy model. This has a number of advantages: (i) It is sufficient to run the folding algorithm only once for the entire alignment, which significantly reduces the computational effort, in particular for larger data sets. (ii) The reliability of prediction can be assessed fairly directly by computing the matrix of base-pairing probabilities instead of the minimum energy structure (or a small ensemble of sub-optimal folds). (iii) If the sequences do not admit a common fold, the method will not predict base-pairs.

Theory

From an algorithmic point of view, RNA secondary structure prediction can be viewed as a (complicated) variant of the maximum circular matching problem (MCMP). We briefly outline the simplified model here to highlight the idea behind the alifold algorithm. The RNA folding problem, with a realistic energy model that is based on extensive thermodynamic measurements, can be solved using a similar
dynamic programming scheme as for the MCMP (Figure 1).

We are given a sequence of nucleotides \( x = (x_1, \ldots, x_n) \) of length \( n \) and energy parameters \( \beta_{ij} \) describing the stability of the base-pair \( (x_i, x_j) \). In the simplest case \( \beta_{ij} = -1 \) for every base-pair that is formed. RNA folding of course has to obey the logic of base-pairing, thus we introduce the pairing matrix \( \Pi \) of the sequence \( x \) with the entries \( \Pi_{ij} = 1 \) if sequence positions \( i \) and \( j \) can form a base-pair, i.e. if \( (x_i, x_j) \) is in the set of allowed base-pairs \( B = \{GC, CG, AU, UA, GU, Ug\} \), and \( \Pi_{ij} = 0 \) if \( x_i \) and \( x_j \) cannot pair. The second important restriction is that a base-pair must span at least \( m = 3 \) unpaired bases, i.e. if \( (i, j) \) is a pair then \( j > i + m \). The RNA version of the MCMP thus consists of finding a secondary structure \( \Sigma \) on \( x \) that contains only allowed base-pairs (\( \Pi_{ij} = 1 \)) and minimizes the total energy \( E = \sum_{i < j} E_{ij} \).

The best energy on the subsequence from position \( i \) to \( j \) is denoted by \( E_{ij} \). Because of the non-crossing rule a base-pair \( (i, k) \) separates the secondary structure into a secondary structure on the sub-sequence from \( i + 1 \) to \( k - 1 \) and a secondary structure from \( k + 1 \) to \( j \). The latter may be empty if \( k = j \), of course. Therefore, \( E_{ij} \) satisfies the following recursion:

\[
E_{ij} = \min \left\{ E_{i,j-1}; \min_{k; n-m < k \leq j} E_{i+1,k-1} + E_{k+1,j} + \beta_{ij} \right\}
\]

(1)

The value \( E_{1,n} \) is the minimal energy for a secondary structure of the sequence \( x \). The (triangular) matrix \( E \) has \( O(n^2) \) entries, and the computation of each entry requires a minimum over \( O(n) \) terms, hence the total effort is \( O(n^3) \). The structure itself, i.e. the list of base-pairs, can be recovered by standard back-tracking from the matrix \( E \).

While \( \beta_{ij} \) depends only on the type of the base-pair \( (x_i, x_j) \) in the usual ansatz there is nothing to prevent us to use a more sophisticated cost function that summarizes all our knowledge on the base-pair, not just its thermodynamic stability. Most importantly, we can use \( \beta_{ij} \) incorporate knowledge about sequence covariations into the folding procedure.

Figure 1. Consensus secondary structure of the 14 SRP RNA of Archea contained in SRPDB\(^8\) (MET.JAN., MET.VOL., MET.FER., MET.THE., MET.ACE., HAL.HAL., ARC.FUL., PYR.ABY., PYR.HOR., THE.CEL., PYR.OCC., AER.PER., SUL.SO-A, SUL.SO-B). We use this example to explain the representation of the results: L.h.s: Mountain plot. A base pair \((i, j)\) is represented by a slab ranging from \(i\) to \(j\). The 5’ and 3’ sides of stems thus appear as up-hill and downhill slopes, respectively, while plateaus indicated unpaired regions. Mountain plots\(^3\) are equivalent to the conventional drawing (r.h.s.) but have the advantage that (1) they can be compared more easily, and (2) it is easier to display additional information about both sequence variation (color code) and thermodynamic likeliness of a base-pair (indicated by the height of the slab and the size of the dot, respectively). Colors in the order red, ocher, green, cyan, blue, violet indicate 1–6 different types of base-pairs. Pairs with one or two inconsistent mutation are shown in (two types of) pale colors. The shaded mountain in the background is the phylogenetic structure taken from the SRPDB. The close match is easily visible. It appear higher because the phylogenetic structure contains base-pairs that correspond to deletions in the majority of the structures and because the height of base-pair in the alifold structure is in general somewhat less than \( p = 1.0 \). R.h.s: In the conventional secondary structure graph paired positions with consistent mutations are indicated by circles around the varying position. Compensatory mutations thus are shown by circles around both pairing partners. Inconsistent mutants are indicated by gray instead of black lettering.
Assume that we are given a multiple sequence alignment $A_{ij}$ of $N$ sequences. By $A_{ij}$, we denote the $ith$ column of the alignment, while $a^{\alpha}_{i}$ is the entry in the $ith$ row of the $jth$ column. The length of $A_{ij}$, i.e., the number of columns, is $n$. Furthermore, let $f_{ij}(X)$ be the frequency of base $X$ at aligned position $i$ and let $f_{i}(XY)$ be frequency of finding $X$ in $i$ and $Y$ in $j$.

The most common way of quantifying sequence covariation for the purpose of RNA secondary determination is the mutual information score $^{25-27}$

$$M_{ij} = \sum_{XY} f_{ij}(XY)\log \frac{f_{ij}(XY)}{f_{i}(X)f_{j}(Y)} \quad (2)$$

Usually, the mutual information score makes no use of RNA base-pairing rules. For large datasets this is desirable, since it allows identifying non-canonical base-pairs and tertiary interaction. For the small datasets considered here, neglecting base-pairing rules does more harm (by increasing noise) than good. In particular, mutual information does not account at all for consistent non-compensatory mutations, i.e. if we have, say, only GC and GU pairs at positions $i$ and $j$ then $M_{ij} = 0$. Thus sites with two different types of base-pairs are treated just like a pair of conserved positions. We argue, however, that the information contained in consistent mutations such as GC$\rightarrow$GU should not be neglected when dealing with sparse datasets that contain too little sequence variation to use phylogenetic methods alone.

As a consequence we prefer a covariance-like measure distinguished between conserved pairs, pairs with consistent mutations, and pairs with compensatory mutations. It is convenient to use the abbreviation:

$$d_{ij}^{\alpha,\beta} = 2 - \delta(a^{\alpha}_{i}, a^{\beta}_{j}) - \delta(a^{\beta}_{i}, a^{\alpha}_{j}) \quad (3)$$

where $\delta(a^{\alpha}_{i}, a^{\alpha}_{j}) = 1$, if $a^{\alpha}_{i} = a^{\alpha}_{j}$ and 0, otherwise. Thus $d_{ij}^{\alpha,\beta} = 0$ if the sequences $\alpha$ and $\beta$ coincide in both aligned positions $i$ and $j$, equal to 1 if they differ in one position, and is 2 if they differ in both positions. In other words, $d_{ij}^{\alpha,\beta}$ is the Hamming distance of the restriction of the sequences $\alpha$ and $\beta$ to the two aligned positions $i$ and $j$.

A straight forward measure of covariation is then

$$C_{ij} = \frac{1}{N} \sum_{\alpha<\beta} d_{ij}^{\alpha,\beta} \Pi_{ij}^{\alpha} \Pi_{ij}^{\beta}$$

$$= \sum_{XY,XY'} f_{ij}(XY)D_{XY,XY'}f_{ij}(XY') \quad (4)$$

where the $16 \times 16$ matrix $D$ has entries $D_{XY,XY'} = d_{ij}(XY,XY')$ if both $XY \in B$ and $XY' \in B$ and $D_{XY,XY'} = 0$, otherwise. Here $d_{ij}(XY,XY')$ is again the Hamming distance of $XY$ and $XY'$. In passing we note that the second form of equation (4) is a scalar product, $C_{ij} = (f_{ij}D_{ij})$, and hence can be evaluated efficiently.

Both the mutual information score and the covariance score give a bonus to compensatory mutation. Neither score deals with inconsistent sequences, i.e. with sequences that cannot form a base-pair between positions $i, j$. The simplest ansatz for this purpose is:

$$q_{ij} = 1 - \frac{1}{N} \sum_{\alpha} \left\{ \Pi_{ij}^{\alpha} + \delta(a^{\alpha}_{i}, \text{gap})\delta(a^{\alpha}_{j}, \text{gap}) \right\} \quad (5)$$

This simply counts the number of inconsistent sequences, where combinations of a nucleotide and a gap are counted as inconsistent while gap-gap combinations (i.e. deletions of an entire base-pair) are ignored.

In a multiple alignment of a larger number of sequences we have to expect one or the other sequencing error and of course there will be alignment errors. Thus, we cannot simply mark a pair of positions as non-pairing if a single sequence is inconsistent. Furthermore, there is the possibility of a non-standard base-pair. $^{27}$ Thus we define a threshold value $B^{*}$ for the combined score $B_{ij} = C_{ij} - \phi_{1}q_{ij}$ and set:

$$\Pi_{ij}^{\alpha} = \begin{cases} 0 & \text{if } B_{ij} < B^{*} \\ 1 & \text{if } B_{ij} \geq B^{*} \end{cases} \quad (6)$$

for the pairing matrix of the alignment. The energy model for the MCMP is then obtained as a linear combination of the average pairing energy and the combined covariation score $B_{ij}$:

$$\beta_{ij}^{\alpha} = \frac{1}{N} \sum_{\alpha} \epsilon(a^{\alpha}_{i}, a^{\alpha}_{j}) - \phi_{2}B_{ij} \quad (7)$$

where $\epsilon(a^{\alpha}_{i}, a^{\alpha}_{j})$ is the pairing energy contribution for a $(a^{\alpha}_{i}, a^{\alpha}_{j})$ pair in sequence $\alpha$. In practice, “loop-based” energy models perform much better. The secondary structure is decomposed into its loops (faces of the planar drawing) and each loop is assigned an energy dependent on loop-type (stacked pairs, hairpin loops, interior loops, multi-branched loops), size, and sequence. Up-to-date parameters for this model are tabulated. $^{23}$ We set the total energy of an alignment-folding as the average of loop-based energies of all sequences plus the covariance contribution.

In addition to the standard energy model for RNA folding we have therefore only the threshold value $B^{*}$ and the two scaling factors $\phi_{1}$ and $\phi_{2}$. Their default values are listed in Table 1. With these values the effect of a compensatory mutation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold for pairing ($B^{*}$)</td>
<td>-1.00</td>
</tr>
<tr>
<td>Relative weight of inconsistent sequences ($\phi_{1}$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Weight of sequence covariation ($\phi_{2}$) (kcal/mol)</td>
<td>1.00</td>
</tr>
</tbody>
</table>
is comparable to the energy gained by extending a helix by one base-pair. In a few tests using three 23 S rRNA sequences, these default values were indeed optimal, while variations within a factor 2 of the default did not have strong effects. In addition, non-standard base-pairs (including gap–gap combinations) can occur in the alignment folding for which no measured energy parameters are available. We substitute the default stacking energy of 0.0 kcal/mol in this case (1 cal = 4.184 J).

The values for $B^*$ and the linear combination coefficients $f_1$ and $f_2$ have to be estimated with the expected values of the covariance term $C_{ij}$ and the non-bonding term $q_{ij}$ for uncorrelated random

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Figure 2. Mountain representation of the secondary structure of *E. coli* rRNAs. Upper panel: 16 S RNA (*A. globiformis, Anabaena* sp., *A. tumefaciens, B. japonicum, E. coli*), lower panel 23 S RNA (*B. subtilis, T. thermophas, Pir. marina, Rh. sphaero, E. coli*). Green line: predicted mfe structure; black line: phylogenetic structure; solid colored area: RNAalifold prediction for *E. coli* from alignment of five sequences.
sequences in mind:

\[
(C_{ij}) = \frac{6 \times 0 + 8 \times 1 + 22 \times 2}{16^2} = \frac{13}{64} \approx 0.203
\]

\[
(q_{ij}) = 1 - \frac{6}{36} = 0.833
\]

(8)

Here the expectation of \( C_{ij} \) is computed for a sample of independent random RNA sequences.

The reliability of thermodynamics-based RNA folding is increased substantially by taking suboptimal structures into account. This can be achieved either by explicitly generating a list of suboptimal structures (as in Zuker’s mfold\textsuperscript{28} or the program RNAsubopt from the Vienna group\textsuperscript{29}) or by directly computing the pairing probabilities for all possible base-pairs. McCaskill’s partition function algorithm\textsuperscript{30} produces the complete matrix \( P \) of pairing probabilities with time and memory requirements comparable to the simpler minimum energy folding. The partition function algorithm is easily extended to work with the modified energy functions in the same way as the minimum energy folding algorithm.

The covariance term (equation (4)) can be biased if the sequences are strongly clustered. A more accurate approach to quantifying the sequence covariances should therefore explicitly account for the phylogenetic relationships of the aligned sequences. A maximum likelihood approach for this task is outlined.\textsuperscript{30} We have experimented with a parsimony-based approach in which covariations are not counted between all pairs of sequences but only along the edges of an inferred phylogenetic tree. It appears, however, that at least for data sets considered in this study the simple covariance term yields equally good results.

### Results and Discussion

Purely phylogenetic methods can be used to derive conserved elements or a consensus structure only when a sufficiently large number of sequences is available, while the accuracy of purely thermodynamic structure prediction is often not satisfactory. In contrast, the alignment folding procedure introduced in this contribution predicts over 80% of the base-pairs correctly from a dataset of only five sequences with an automatically generated alignment, as the examples in Figure 2 show, see also Table 2.

The consensus structure of a set of RNA sequences has to be distinguished from the collection of structural features that are conserved. Whenever there are reasons to assume that the structure of the whole molecule is conserved one may attempt to compute a consensus structure. On the other hand, consensus structures are unsuitable when a significant part of the molecule has no conserved structures. RNA virus genomes, for instance contain only local structural patterns (such as the IRES in picornaviruses or the TAR hairpin in HIV). Such features can be identified with a related approach that is implemented in the algorithms alidot and pfrali\textsuperscript{17,18} and requires structure prediction for each individual sequence. The automatic search for conserved structures should not return false positives and hence has been designed not to predict secondary structures at all unless structure is unambiguously preserved among the sequences. For small sets of sequences pfrali therefore predicts only about half of the base-pairs of the phylogenetic structure and leaves out regions with little sequence variation and ambiguous thermodynamic structure predictions (data not shown).

In Figure 3 we compare the RNAalifold consensus structure with the conserved parts of the structure as predicted by pfrali for the mRNAs of the early nodulin gene enod40 from nine plant species. Enod40, which is coding for an RNA of about 700 nt, is expressed in the nodule primordium developing in the root cortex of leguminous plants after infection by symbiotic bacteria. Translation of two sORFs (I and II, 13 and 27 amino acid residues, respectively) present in the conserved 5’ and 3’ regions of enod40 was required for this biological activity.\textsuperscript{31}

A stem-loop structure located just after the first ORF is proposed.\textsuperscript{31} Its location, indicated by a narrow bar in Figure 3, coincides with a signal in the pfrali prediction but does not appear in the RNAalifold consensus structure. A comparison of

### Table 2. Quality of predictions

<table>
<thead>
<tr>
<th>E. coli 16S RNA</th>
<th>RDB</th>
<th>ClustalW</th>
<th>Filled</th>
<th>Raw</th>
<th>Filled</th>
<th>Raw</th>
<th>Filled</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Raw</td>
<td>Filled</td>
<td>Raw</td>
<td>Filled</td>
<td>Raw</td>
<td>Filled</td>
</tr>
<tr>
<td>1</td>
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<td>N/A</td>
<td>47.2</td>
<td>N/A</td>
<td>52.2</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>64.7</td>
<td>67.1</td>
<td>73.8</td>
<td>73.4</td>
<td>71.0</td>
<td>69.4</td>
</tr>
<tr>
<td>3</td>
<td></td>
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<td>77.2</td>
<td>78.1</td>
<td>79.9</td>
<td>71.2</td>
<td>73.7</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>74.5</td>
<td>81.2</td>
<td>85.2</td>
<td>86.6</td>
<td>76.2</td>
<td>82.4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>74.7</td>
<td>82.1</td>
<td>85.9</td>
<td>86.6</td>
<td>74.6</td>
<td>82.6</td>
</tr>
</tbody>
</table>

Data are compared for two alignments and different number \( N \) of aligned sequences, both for the raw RNAalifold prediction and the filled-in structure (refer to text).
Figure 3. Mountain plots for nine enod40 sequences (PSENOD40, TRJ00268, MSENOD40, MTENOD40R, MSENOD40R, AF013594, PVENOD40, GMENOD401, SRENOD40) taken from the database. The short ORF is marked by a gray bar. L.h.s.: consensus structures from RNAalifold; r.h.s: pfali prediction. Both methods unambiguously detect a stem-loop structure (alignment positions 272–450), and the hairpin structure (468–500) which is located within the longer ORE II. The structure (156–190) partially overlaps with ORF I; it is not well predicted by pfali. The location of the putative RNA secondary structure described (in Figure 7) is marked by the narrow bar.
Materials and Methods

Sequence data were retrieved from publicly accessible RNA databases: the SRPDB,\(^\dagger\) the non-coding RNA database,\(^\dagger\) and the Ribosomal Database Project.\(^\dagger\) The \textit{E. coli} rRNA reference secondary structures were retrieved from Robin Gutell’s Comparative RNA Web Site. These structures are generated from covariation data alone and manually refined; the \textit{E. coli} structure represents the standard model of rRNA structure. Non-standard base-pairs and pseudo-knots pairs were removed for comparison with predicted structures. The pfrali program, on the other hand, picks up the few conserved pairs and reports a structural element with many “holes”.

Both methods agree on a number of other conserved secondary structure elements in \textit{enod40} RNAs that are supported by a significant number of sequence covariations. Whether some or all of these structural features are functional is unknown at present. One likely possibility is that they might take part in localization of mRNA translation.\(^\dagger\)

Acknowledgments

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